



**KAPITAŁ LUDZKI**  
NARODOWA STRATEGIA SPÓJNOŚCI

**UNIA EUROPEJSKA**  
EUROPEJSKI  
FUNDUSZ SPOŁECZNY



## **BIOPHYSICS**

**Prezentacja multimedialna współfinansowana przez  
Unię Europejską w ramach  
Europejskiego Funduszu Społecznego w projekcie pt.  
*„Innowacyjna dydaktyka bez ograniczeń - zintegrowany  
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do zatrudniania osób niepełnosprawnych”***



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# Transport (4)

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Technical University of Lodz*





## Membrane Transport Systems - general remarks

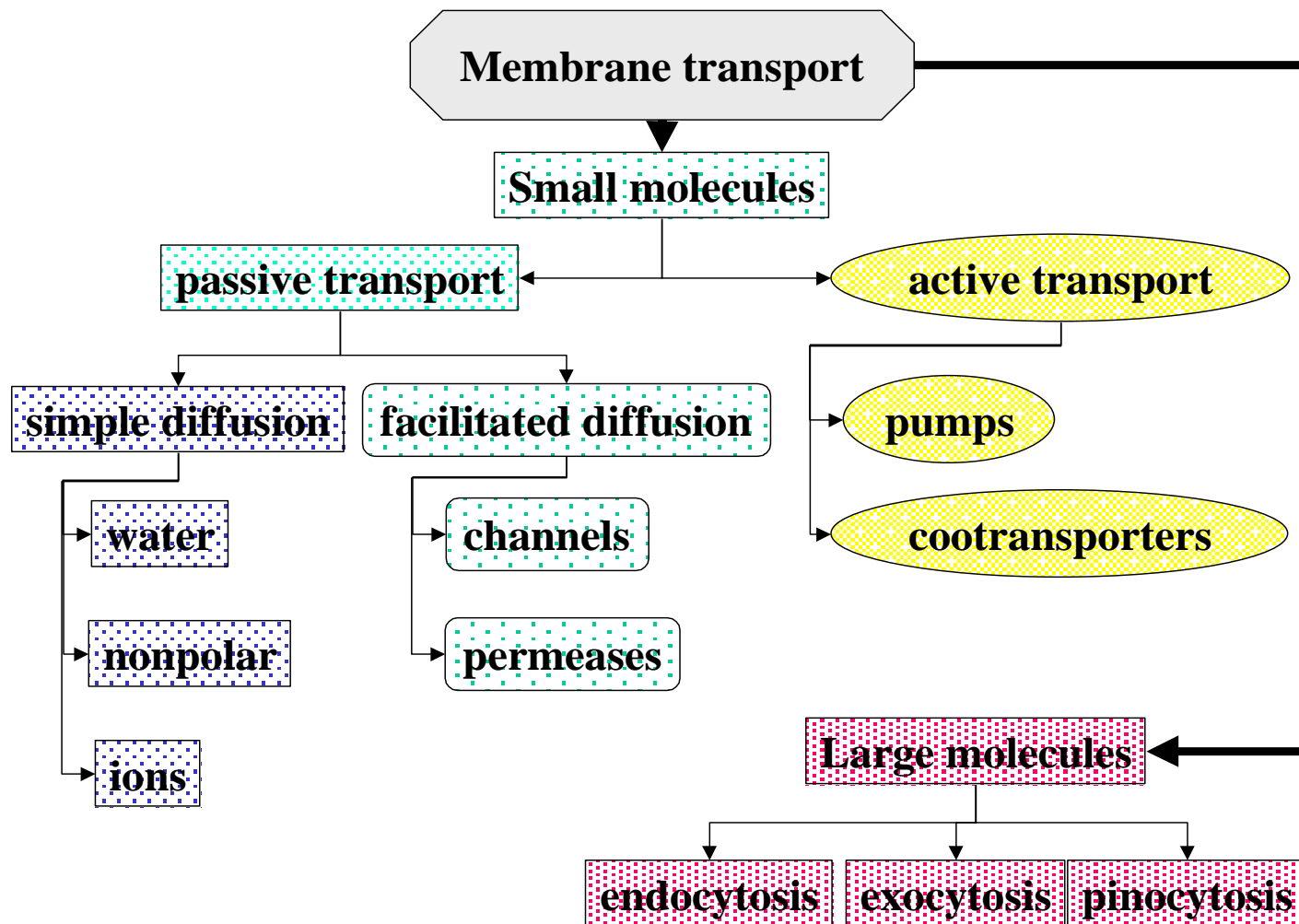
Transport of nutrients, ions, and excretory substances from one side to the other is a major function of the cell membrane.

A number of different systems have been developed to fulfill this function.

Generally, the permeation of small molecules across the membrane is quite different from molecules too large to penetrate membrane.



# Membrane Transport Systems - general remarks





# Transport of Small Molecules - Passive transport

Depending on whether a cell pays for the transport (energetically) we talk about **passive** (free) and **active** transport

Passive transport:

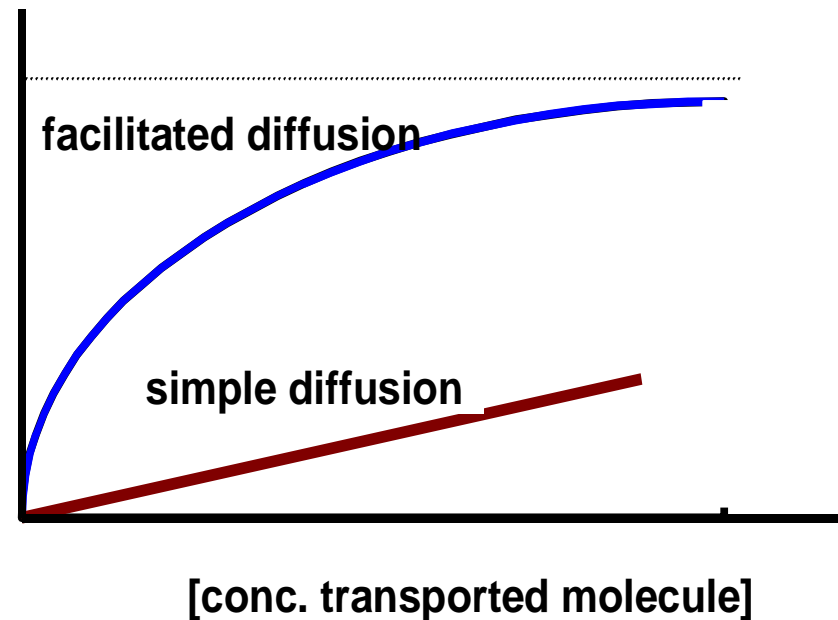
**Simple diffusion** - the simplest form of transport is passive diffusion. It does not need any helpers.

**Facilitated diffusion** - Some molecules diffuse freely but with the help of another molecule.

Passive transport is always driven by a concentration gradient and undergo to Fick's Law:

$$dn/dt = -DS (dC/dL)$$

rate of transport





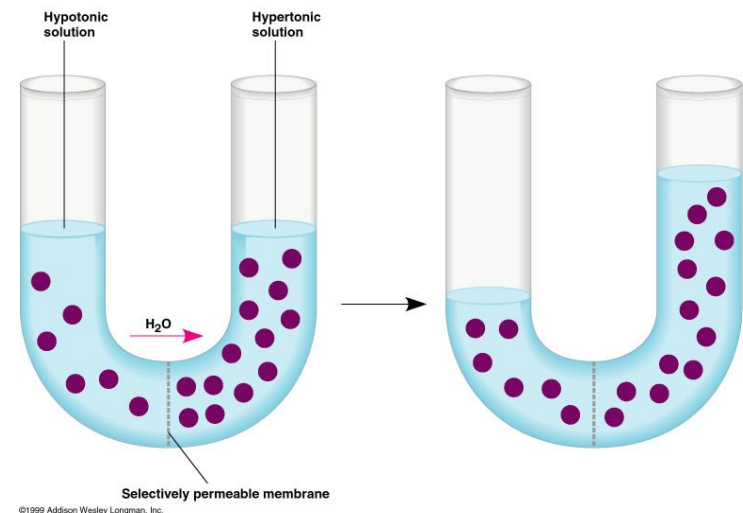
# Transport of Small Molecules; Water Diffusion - Osmosis

Lipid membranes are **semi-permeable**; some substances pass through freely (**water**) some don't (**ions**)

Consider two water solutions, one rich in ions and the other not, which are separated by a semi-permeable membrane. Water can move across the membrane in both directions, but because ions attract water and impede its random diffusion, water is retarded on the ion-rich side, therefore the rate from the ion-rich side is less than the rate of ions permeating the membrane from the other side.

The **net movement** of water toward the ion-rich solution builds up hydrostatic pressure, called **osmotic pressure**, which at some point will counteract the attraction of ions.

The two sides will then be at equilibrium.

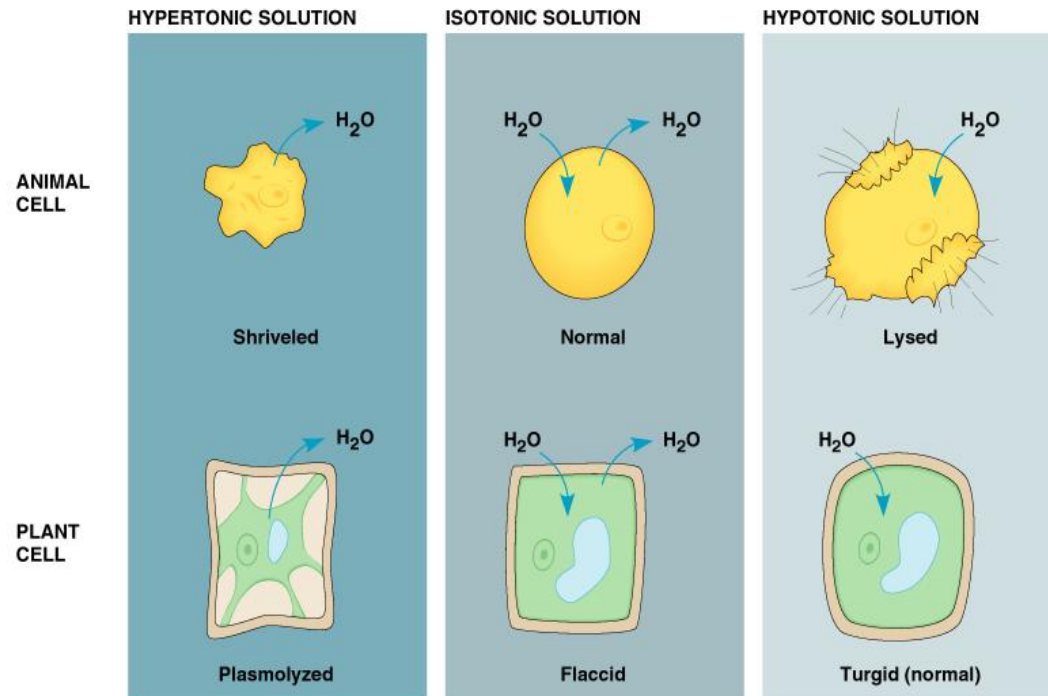


# Transport of Small Molecules; Water Diffusion - Osmosis

We say, the more concentrated solution is **hypertonic** with respect to solution less rich in the impermeant substance. The water will always try to **rush in** to make the more concentrated solution less hypertonic.

The less concentrated solution is referred to as **hypotonic**, water will attempt to **leave** this compartment and thereby decrease concentration of impermeant solute.

When two compartments are equally concentrated they are **isotonic** with respect to each other, and there is **no net diffusion** of water.



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# Transport of Small Molecules; Uncharged Molecules Diffusion

Gases like  $\text{CO}_2$ ,  $\text{O}_2$ ,  $\text{N}_2$ , diffuse easily through membrane because they have no charge (partial or complete) to interact with water.

Hydrophobic molecules (**oils**) have also no trouble permeating membrane.

Ions do **not penetrate** because of charge and the solvation layer that would have to diffuse with them.





## Transport of Small Molecules; Facilitated Diffusion

Some molecules diffuse freely but with **the help of another molecules**.

In facilitated diffusion, however, the rate is limited by the availability of the helper molecules (**carriers - channels or permeases**). Once all the helpers are saturated, the increasing concentration of diffusing molecules will only increase a waiting line for the helper and will not increase rate of transport.

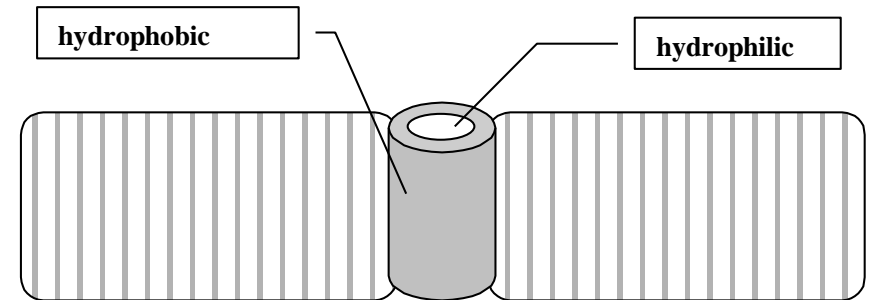
Such a **saturation kinetics** is characteristic of any event (transport, chemical reaction) that requires the help of other molecules.



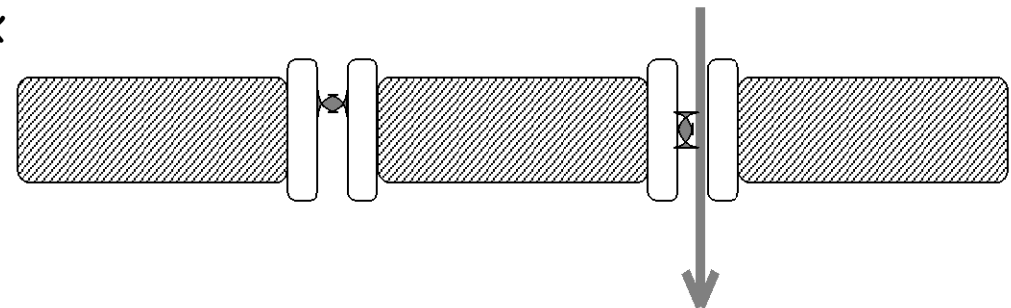
# Transport of Small Molecules; Protein Channels

The simplest form of a helper-facilitator is an **ion channel**.

Channels are usually simple peptides or small proteins of which the outside surface is hydrophobic and the inside hydrophilic, e.g. **gramicidin**.



Gated channel



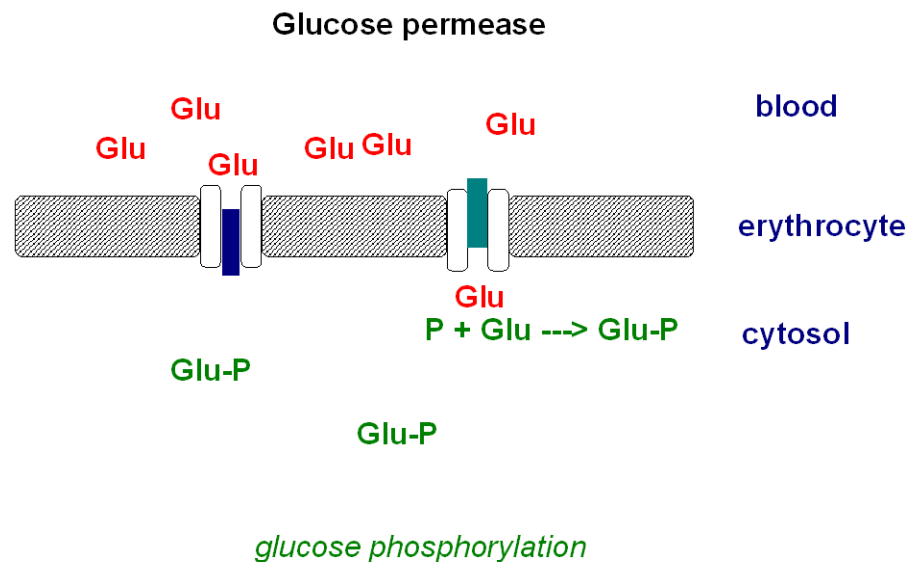
Some channels are more complex they have gates that open in response to a chemical (phosphorylation) or electrical (depolarization) stimulus.

Source: INTERNET

# Transport of Small Molecules; Protein Permeases

More complex than channels are **carrier proteins** such as **glucose permease** in erythrocytes.

The transported molecule (glucose) moves down its concentration gradient. Once inside the cell, the molecule is transformed into another, impermeant species, thus lowering the inside concentration and maintaining the concentration gradient.

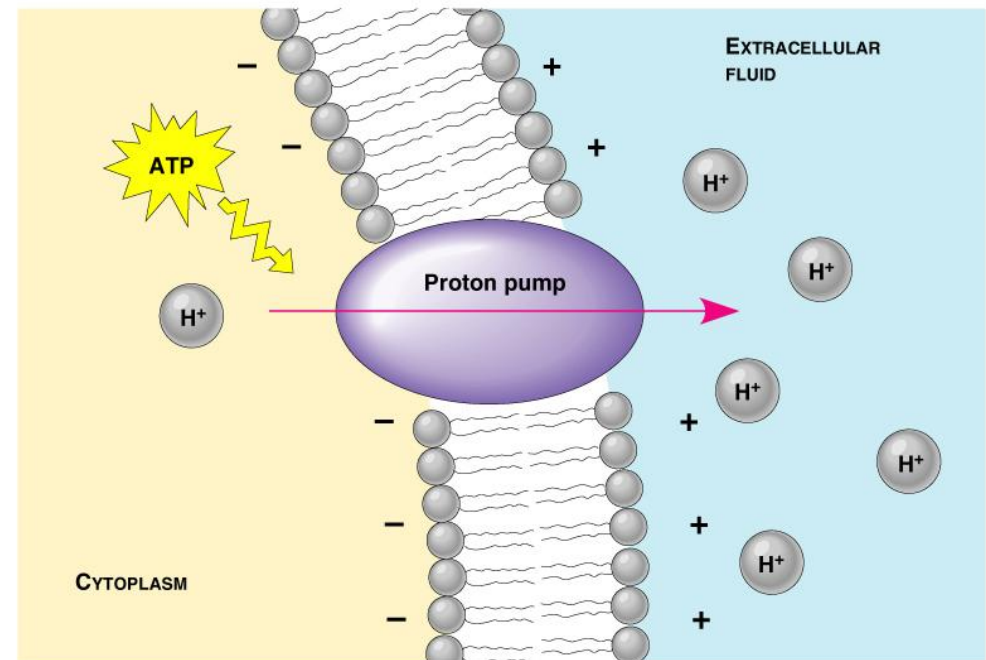


Source: INTERNET

# Transport of Small Molecules; Active Transport

Often the transport has to happen in the direction opposite to the concentration gradient. In order to accomplish this, membranes have mechanisms able to pump the substance from the area of smaller concentration to a compartment with higher concentration. All these schemes cost the cell energy and thus are called **active transport**.

This pump is an ATPase, which means that the enzyme derives its energy from the hydrolysis of ATP.

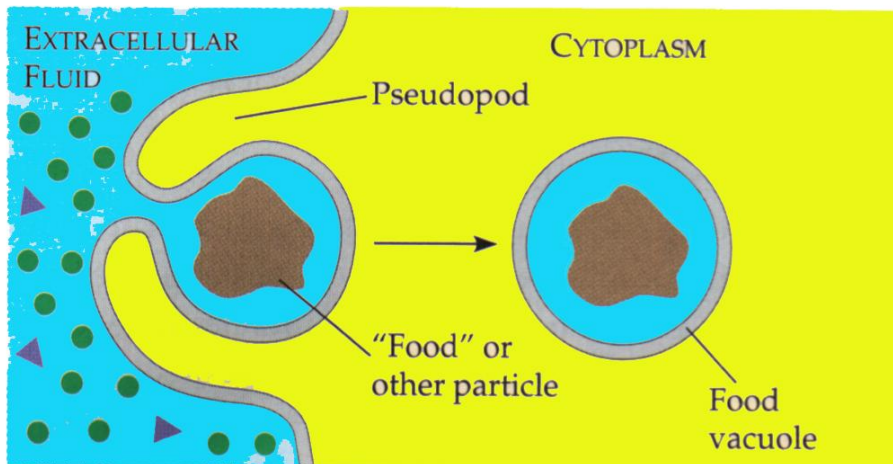


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# Transport of Large Molecules; Phagocytosis

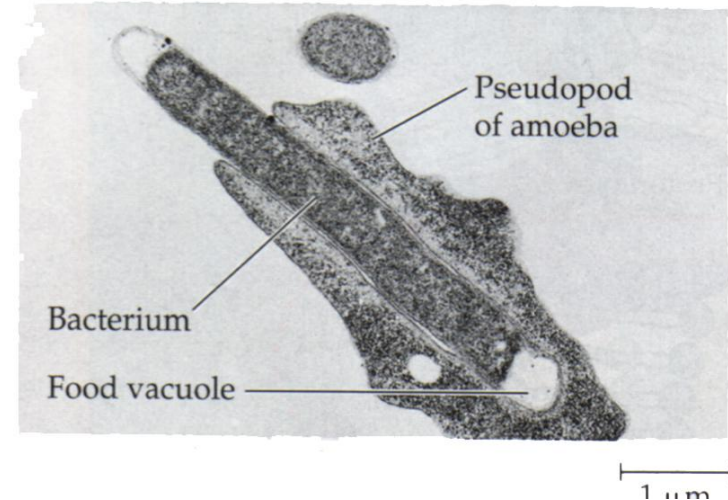
Removal of foreign materials or dead cells by immune cells is a form of endocytosis.

For example, **phagocytes** are **macrophages** that line blood channels of liver (spleen) and eat up aging rbc's; **monocytes** penetrate inflamed tissue and remove the invading bacteria.



(a) Phagocytosis

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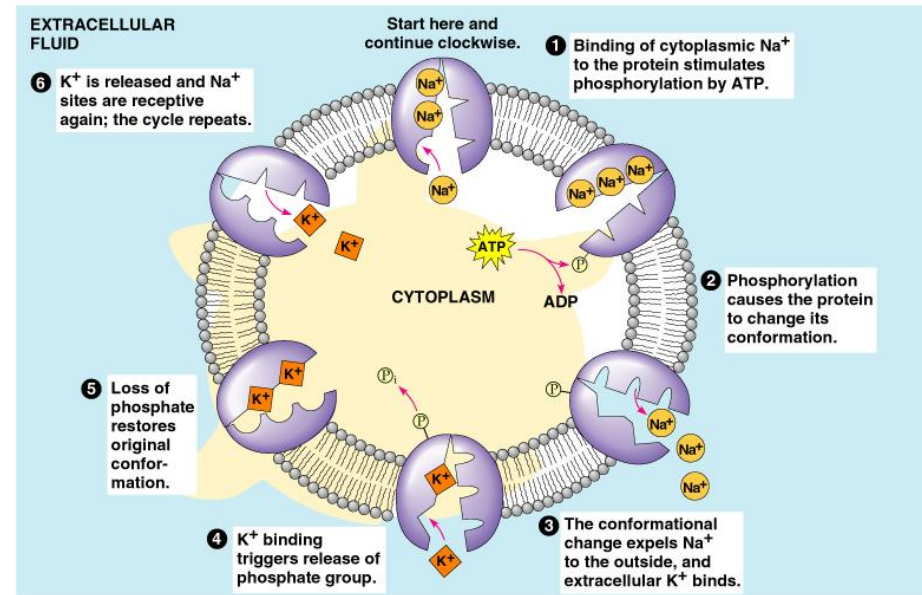


# Transport of Small Molecules; Active Transport

The function of Na/K ATPase is to **set up the electrochemical gradient** of the membrane. It does so by **pumping  $\text{Na}^+$  out** of the cell and pumping  **$\text{K}^+$  into** the cell.

The net effect is to create a chemical potential consisting of two concentration gradients (for Na and for K), as well as electrical potential because **three positive** charges are pumped out while **two positive** charges are pumped in. A negative potential inside the cell is thus created.

**Mechanism:** inside of the cell,  $\text{Na}^+$  binding triggers phosphorylation by ATP; conformational change and  $\text{Na}^+$  release to outside of the cell;  $\text{K}^+$  binding triggers dephosphorylation; inversion to inside of the cell;  $\text{K}^+$  release.





# Transport of Small Molecules; Coupled Transport

Two molecules travel together, one as a **passenger**, the other as a **driver**. The driver diffuses down its electrochemical gradient, but it cannot do so unless it has the passenger.

ATP is not directly involved, but it sets up the electrochemical gradient used to propel the driver.

## Symport

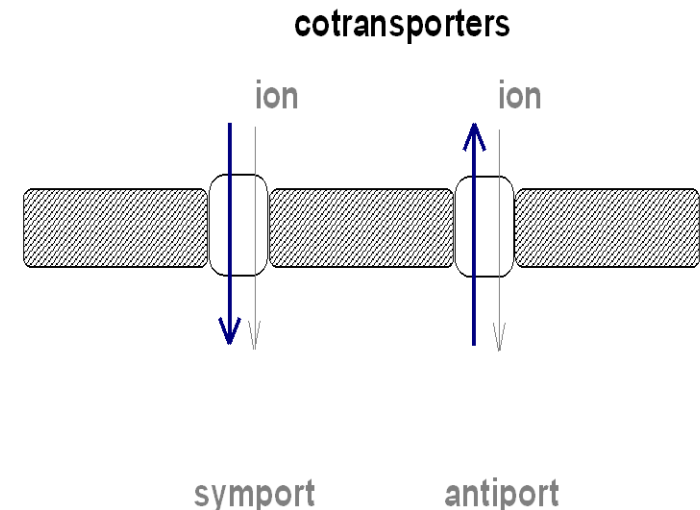
The passenger and the driver are transported in **the same direction**. Na-glucose symport takes place in the intestine, from the gut lumen to the insides of the cells lining the gut.

## Antiport

The driver and passenger travel in **opposite directions**.

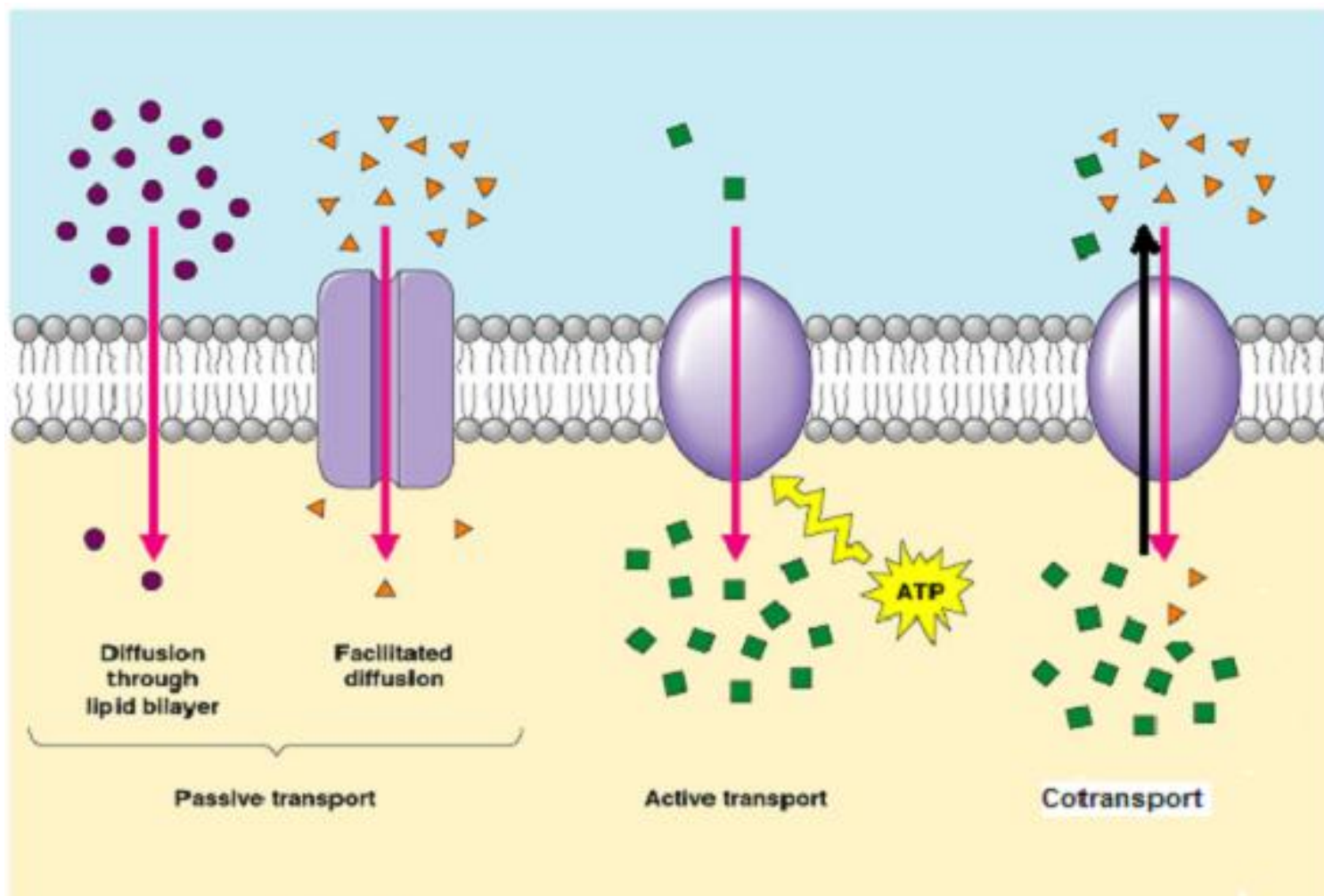
Ca-Na antiport takes place in cardiac muscle.

Very popular are proton-driven pumps: Na-H antiport, Ca-H antiport, sucrose-H antiport in plant vacuoles.



Source: INTERNET

# Transport of Small Molecules; Short Summary



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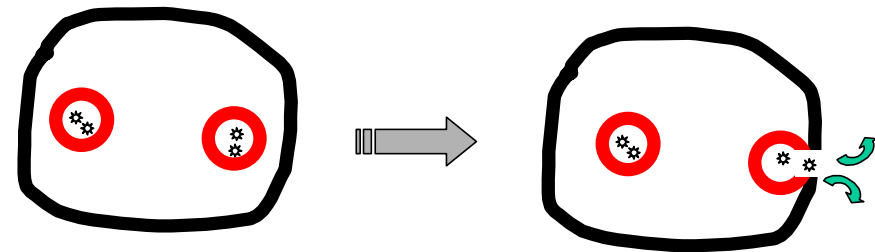
# Transport of Large Molecules; Exo- and Endocytosis

Membranes transport molecules too big to permeate the membrane by engulfing the substance and forming internal vesicles.

Uptake of substances by such a mechanism is called **endocytosis**; the secretion is called **exocytosis**.

In exocytosis, the transport vesicle fuses with the plasma membrane, making the inside of the vesicle continuous with the outside of the cell.

**Exocytosis** is used in secretion of protein hormones (insulin), serum proteins, extracellular matrix (collagen).



**exocytosis**

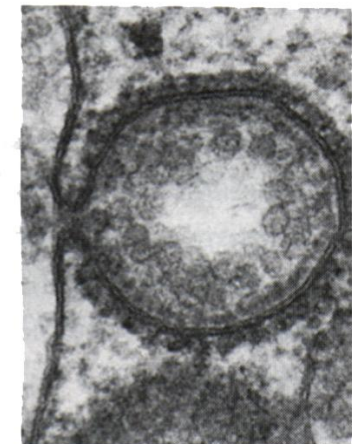
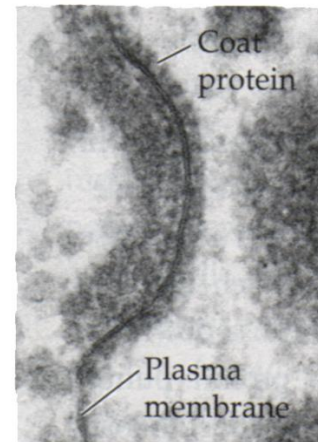
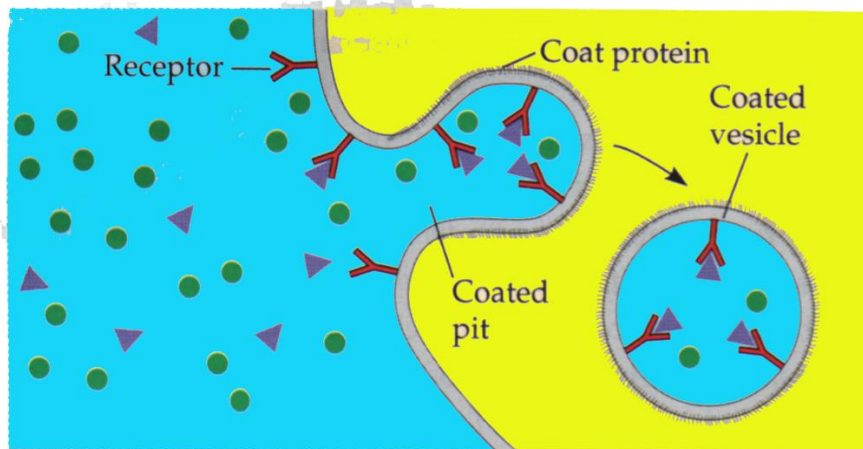
Source: INTERNET

# Transport of Large Molecules; Endocytosis

**Endocytosis** occurs mainly in animal cells, as plants have rigid cell walls.

**Mechanism:** the cell forms pseudopodia that engulf macromolecules; the pseudopodia fuse, and the membrane pinches off, forming an internal vesicle; the vesicle fuses with the lysosome; release.

## Receptor-mediated endocytosis

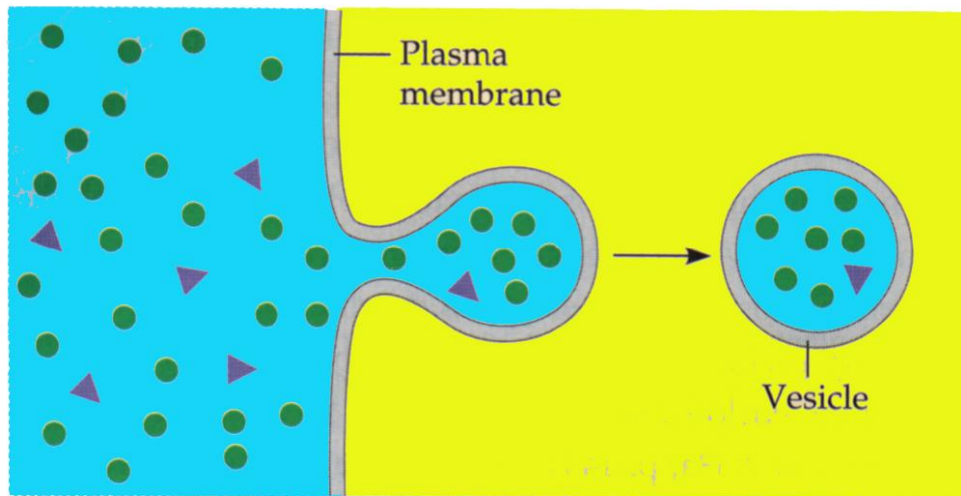


(c) Receptor-mediated endocytosis

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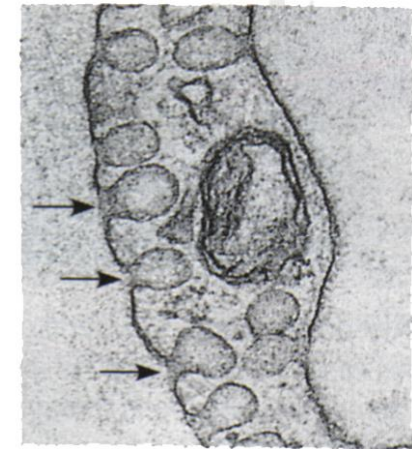
# Transport of Large Molecules; Pinocytosis

Pinocytosis is a **nonspecific uptake** of extracellular solution. Whatever is in the solution is taken up by the cell.



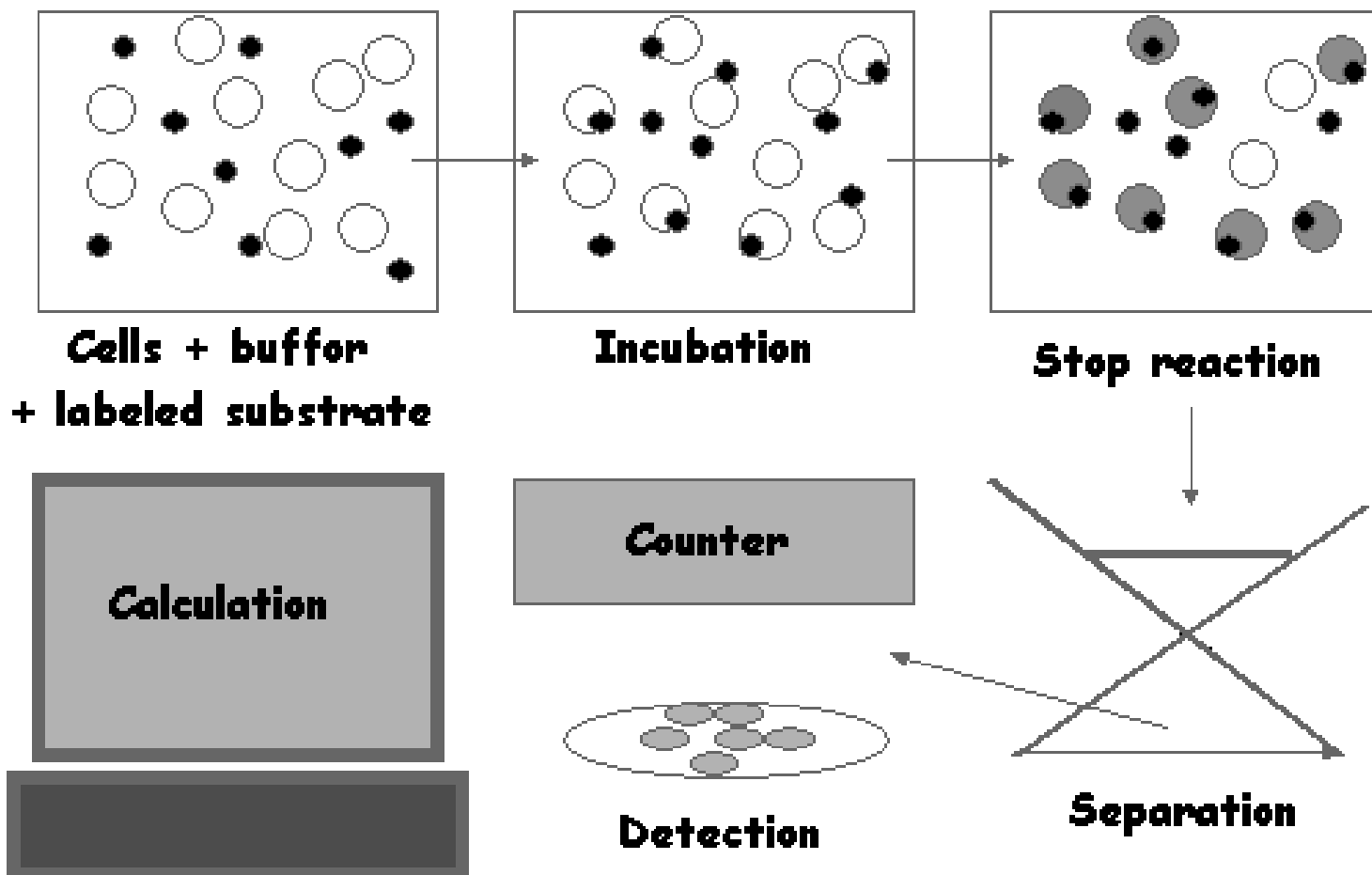
(b) Pinocytosis

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0.5 μm

# Experimental Examination of Transport Systems





# Mathematical Modelling of Amino Acid Transport

Gen. Physiol. Biophys. (1988), 7, 475—494

475

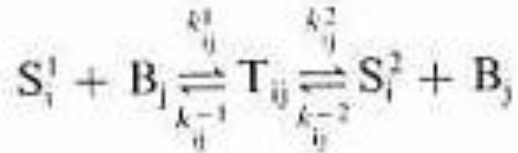
## Mathematical Modelling of the Transport of Low Molecular Weight Solutes Across Biological Membranes. The Transport of Leu, His and Glu into Human Blood Platelets

B. WALKOWIAK and C. S. CIERNIEWSKI

*Department of Biophysics, Institute of Physiology and Biochemistry,  
Medical School of Lodz, 90—131 Lodz, Poland*



# Mathematical Modelling of Amino Acid Transport



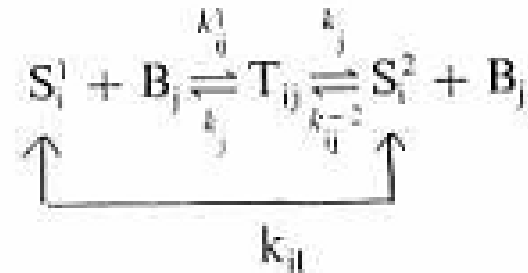
$$k_{ij}^1 = Na_1^+ k_{ij}^*$$

$$k_{ij}^{-2} = Na_2^+ k_{ij}^*$$

$$k_{ij}^1 \gg k_{ij}^{-2} \quad j = 1, \dots, r-1$$

$$k_{ij}^1 \gg k_{ij}^{-2} \quad j = r, \dots, p-1$$

$$k_{ij}^{-1} = k_{ij}^2 = k_j \quad j = 1, \dots, p-1$$



$$i = 1, \dots, m; j = 1, \dots, p-1; l = p, \dots, n$$



# Mathematical Modelling of Amino Acid Transport

$$\frac{dB_j(t)}{dt} = -B_j(t) \sum_{i=1}^m (k_{ij}^1 S_i^1 + k_{ij}^{-2} S_i^2(t)) + 2k_j \sum_{i=1}^m T_{ij}(t)$$

$$\frac{dS_i^2(t)}{dt} = \sum_{j=1}^{p-1} (k_j T_{ij}(t) - B_j(t) k_{ij}^{-2} S_i^2(t)) + \sum_{l=p}^n k_{il} (S_i^1 - S_i^2(t))$$

$$\frac{dT_{ij}(t)}{dt} = B_j(t) (k_{ij}^1 S_i^1 + k_{ij}^{-2} S_i^2(t)) - 2k_j T_{ij}(t)$$

It can be easily seen that the system (6—8) includes first integrals

$$B_j(t) + \sum_{i=1}^m T_{ij}(t) = B_j^0$$

# Mathematical Modelling of Amino Acid Transport

$$\frac{dB_j(t)}{dt} = -B_j(t) \sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j(B_j^0 - B_j(t))$$

The solution to this equation is

$$B_j(t) = B_j^0 \frac{\sum_{i=1}^m k_{ij}^1 S_i^1 \exp\left(-\left(\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j\right)t\right) + 2k_j}{\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j}$$

Substituting Eq. (11) into Eq. (8) we obtain

$$T_{ij}(t) = \frac{k_{ij}^1 S_i^1 B_i^0}{\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j} \left(1 - \exp\left(-\left(\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j\right)t\right)\right)$$



# Mathematical Modelling of Amino Acid Transport

$$S_i^2(t) = S_i^{02} \exp\left(-\sum_{l=p}^n k_{il} t\right) + S_i^1 \left(1 - \exp\left(-\sum_{l=p}^n k_{il} t\right)\right) +$$

$$+ \sum_{j=1}^{p-1} \left[ \frac{k_{ij}^1 k_j S_i^1 B_j^0 \left(\exp\left(-\left(\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j\right)t\right) - \exp\left(-\sum_{l=p}^n k_{il} t\right)\right)}{\left(\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j\right) \left(\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j - \sum_{l=p}^n k_{il}\right)} + \frac{k_{ij}^1 k_j S_i^1 B_j^0 \left(1 - \exp\left(-\sum_{l=p}^n k_{il} t\right)\right)}{\sum_{l=p}^n k_{il} \left(\sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j\right)} \right]$$

$$S_i^{2u}(t) = S_i^{02} \exp\left(-\sum_{l=p}^n k_{il} t\right) +$$

$$+ \left( \sum_{l=p}^n k_{il} S_i^1 + \sum_{j=1}^{p-1} \frac{k_{ij}^1 k_j S_i^1 B_j^0}{2k_j + \sum_{i=1}^m k_{ij}^1 S_i^1} \right) \left( \frac{1 - \exp\left(-\sum_{l=p}^n k_{il} t\right)}{\sum_{l=p}^n k_{il}} \right)$$

# Mathematical Modelling of Amino Acid Transport

$$\frac{dS_i^{2u}(t)}{dt} = \sum_{l=p}^n k_{il} (S_i^l - S_i^{02}) \exp\left(-\sum_{l=p}^n k_{il} t\right) + \sum_{j=1}^{p-1} \frac{k_{ij}^1 k_j S_i^l B_j^0 \exp\left(-\sum_{l=p}^n k_{il} t\right)}{2k_j + \sum_{i=1}^m k_{ij}^1 S_i^l}$$

$$V_i = \frac{dS_i^{2u}}{dt} = K_{Di} (S_j^l - S_i^{02}) + \sum_{j=1}^{p-1} \frac{V_{maxij} S_i^l}{\sum_{i=1}^m S_i^l + K_{Mij}}$$

where  $V_i$  is the rate of concentration change of the  $i$ -th solute inside the cell;

$$V_{maxij} = k_j B_j^0 \exp\left(-\sum_{l=p}^n k_{il} t\right)$$

is the maximal transport rate mediated by the  $j$ -th system;

$$K_{Mij} = \frac{2k_j}{k_{ij}^1}$$

is the Michaelis constant for the  $j$ -th system, and

$$K_{Di} = \sum_{l=p}^n k_{il} \exp\left(-\sum_{l=p}^n k_{il} t\right)$$

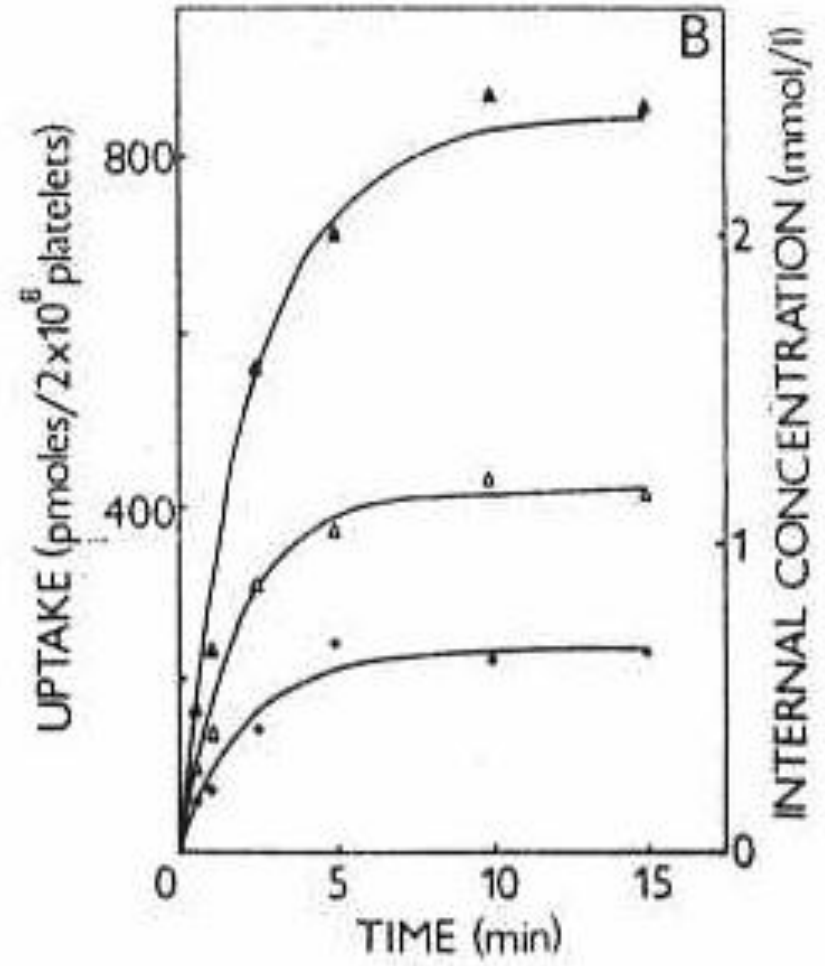
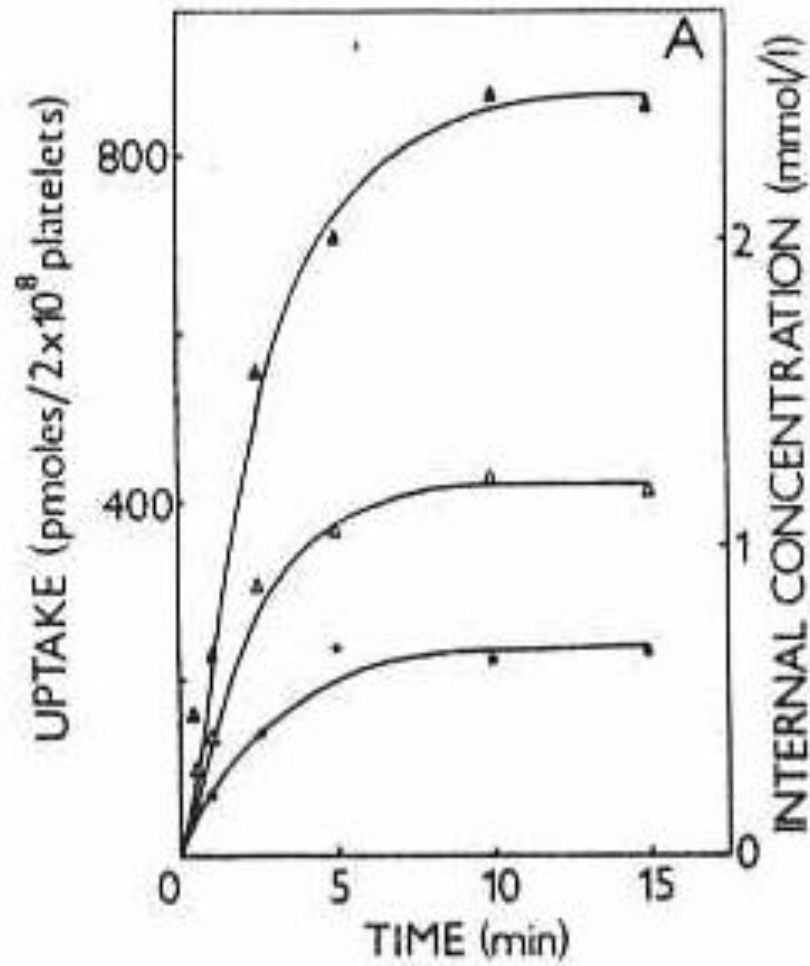
# Mathematical Modelling of Amino Acid Transport

$$\lim_{t \rightarrow \infty} \frac{dS_i^2(t)}{dt} = \lim_{t \rightarrow \infty} \frac{dS_i^{2u}(t)}{dt} = 0$$

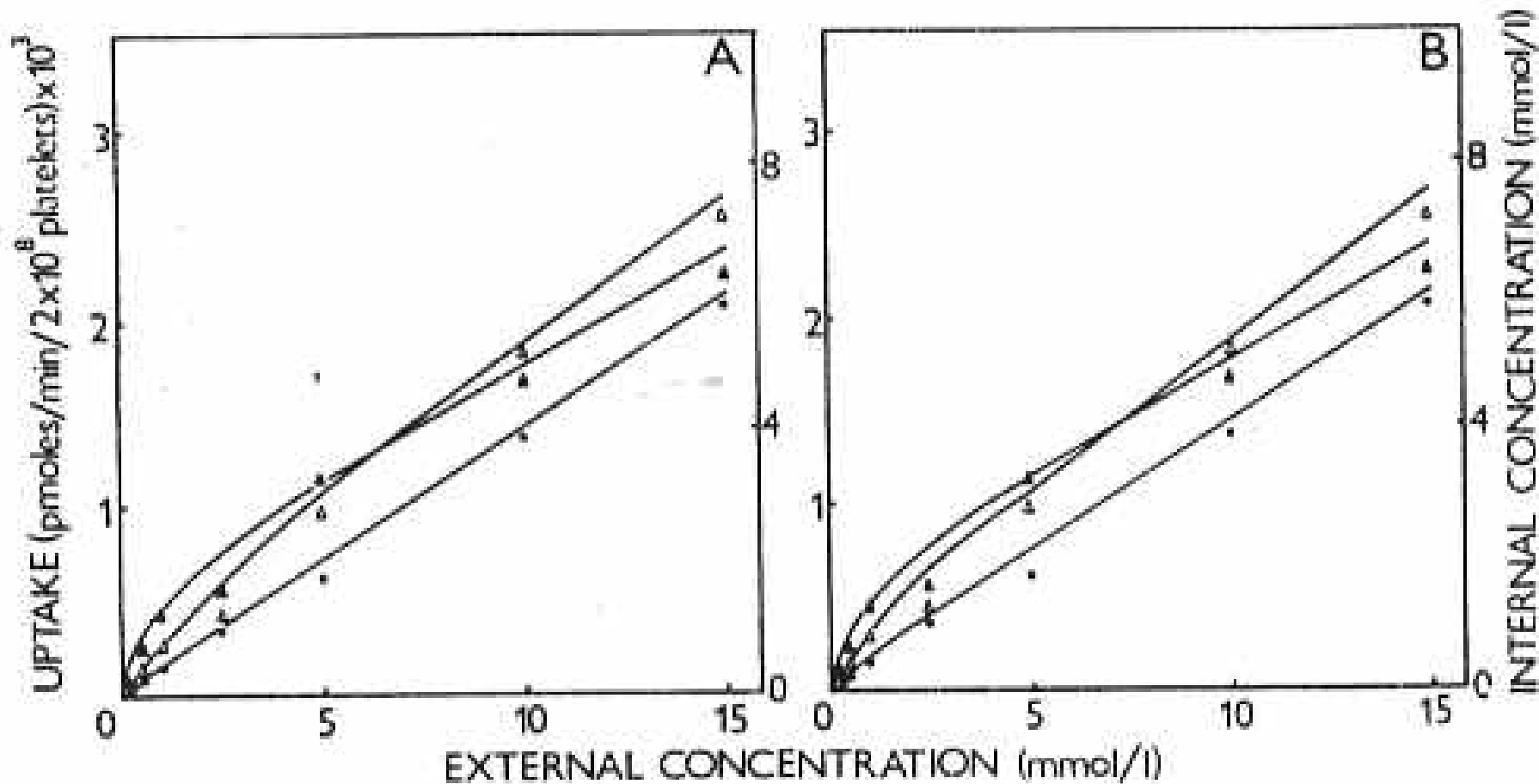
$$\lim_{t \rightarrow \infty} S_i^{2u}(t) = \lim_{t \rightarrow \infty} S_i^2(t) = S_i^1 + \sum_{j=1}^{p-1} \frac{k_{ij}^1 k_j S_i^1 B_j^0}{\left( \sum_{i=1}^m k_{ij}^1 S_i^1 + 2k_j \right) \sum_{l=p}^n k_{il}}$$

$$\sum_{l=p}^n k_{il} (S_i^{02} - S_i^1) = \sum_{j=1}^{p-1} \frac{k_{ij}^1 k_j S_i^1 B_j^0}{2k_j + \sum_{i=1}^m k_{ij}^1 S_i^1}$$

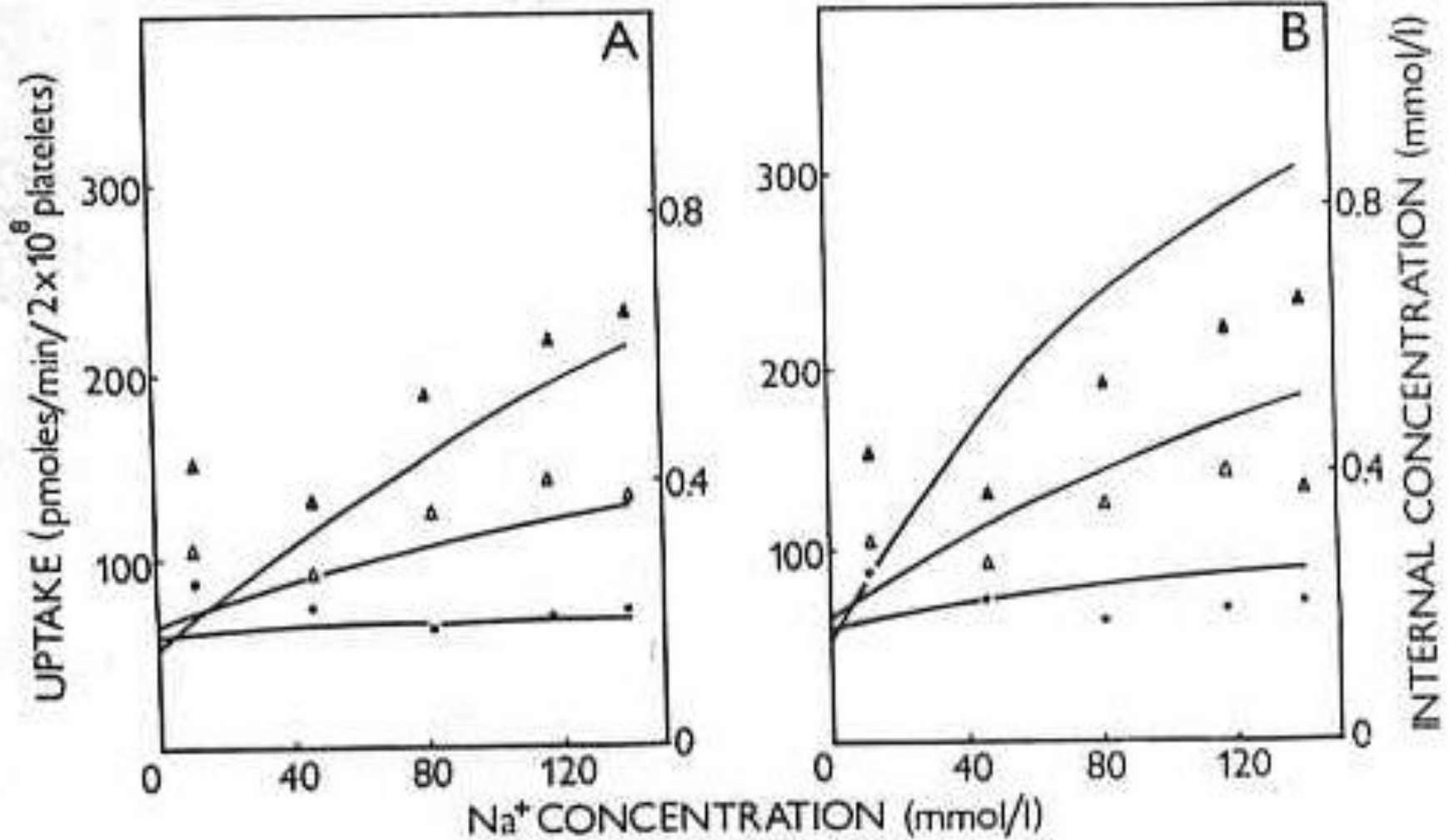
# Mathematical Modelling of Amino Acid Transport



# Mathematical Modelling of Amino Acid Transport

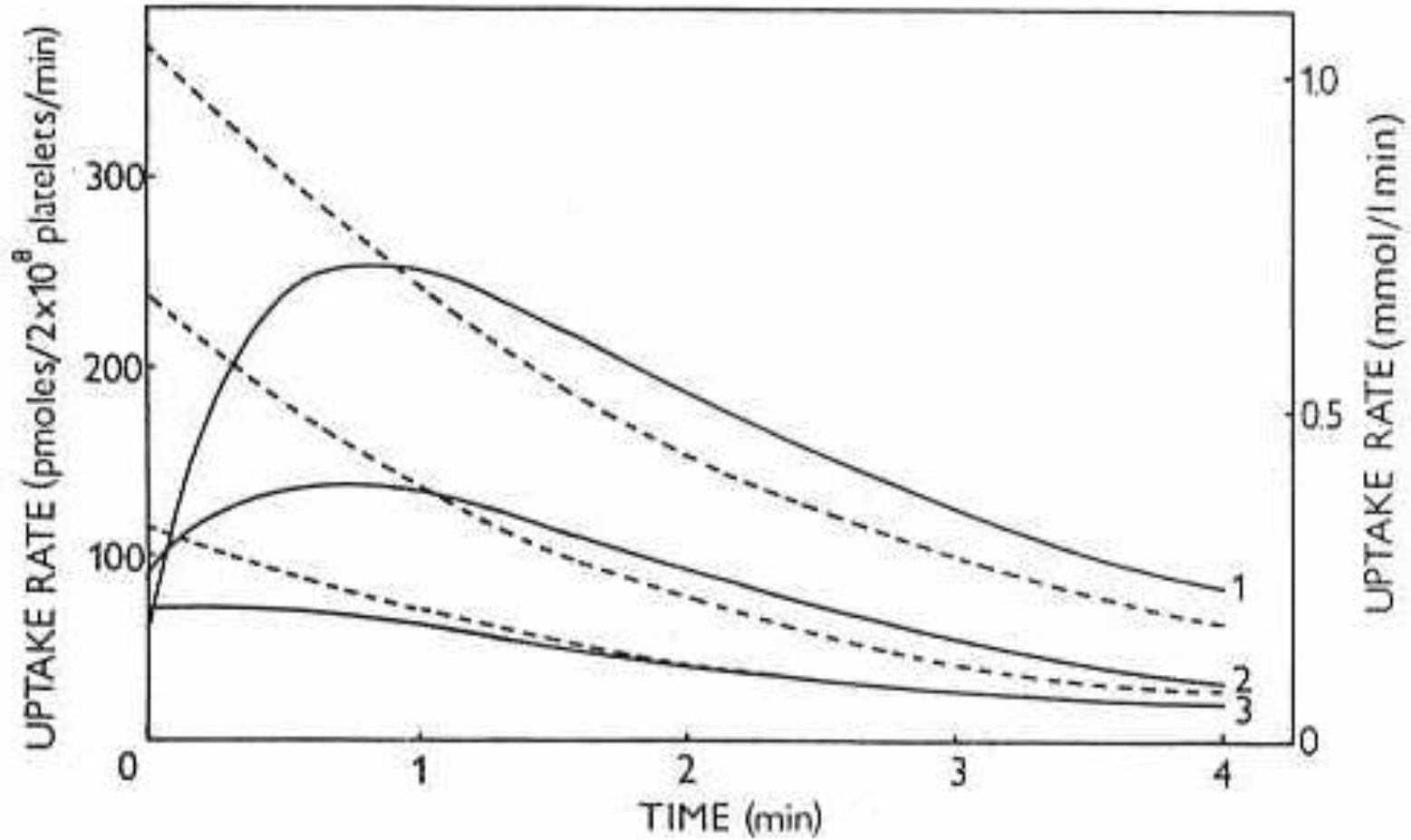


# Mathematical Modelling of Amino Acid Transport





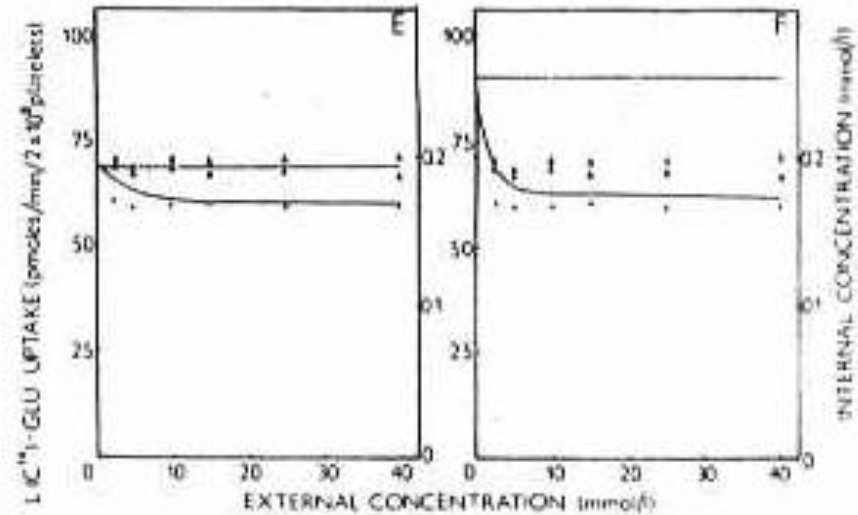
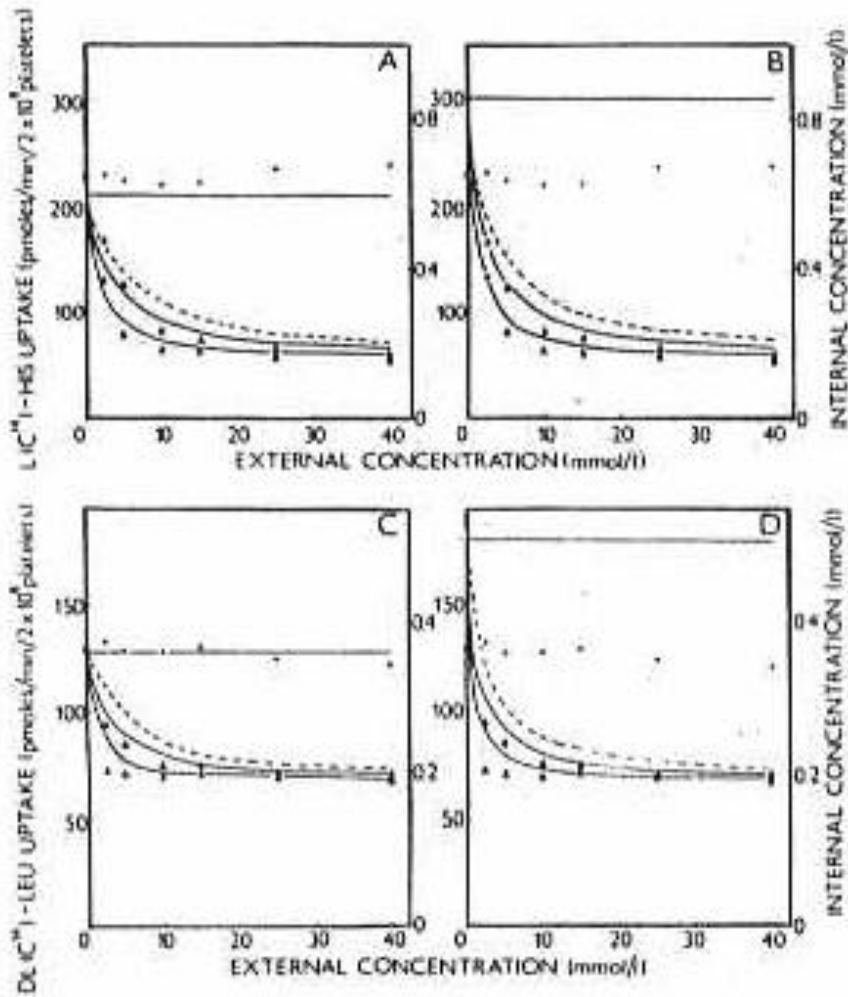
# Mathematical Modelling of Amino Acid Transport







# Mathematical Modelling of Amino Acid Transport





# Mathematical Modelling of Amino Acid Transport

	L-Leu	L-His	L-Glu
exact solution			
$k_{11}^1$	$3.92 \times 10^{-2}$	$4.20 \times 10^{-2}$	$1.22 \times 10^{-2}$
$k_1$	$8.92 \times 10^{-3}$	$9.58 \times 10^{-3}$	$2.93 \times 10^{-3}$
$k_{12}$	$9.65 \times 10^{-3}$	$7.18 \times 10^{-3}$	$8.20 \times 10^{-3}$
$B_1^0$	2.512	3.142	1.368
$R$	0.987	0.989	0.982
simplified solution			
$k_{11}^1$	$3.90 \times 10^{-2}$	$4.19 \times 10^{-2}$	$1.22 \times 10^{-2}$
$k_1$	$8.90 \times 10^{-3}$	$9.52 \times 10^{-3}$	$2.92 \times 10^{-3}$
$k_{12}$	$9.63 \times 10^{-3}$	$7.45 \times 10^{-3}$	$8.13 \times 10^{-3}$
$B_1^0$	2.502	3.116	1.362
$B$	$2.7 \times 10^4$	$3.3 \times 10^4$	$1.5 \times 10^4$
$K_D$	2.59	2.12	2.28
$K_M$	0.456	0.454	0.479
$V_{\max}$	5.97	8.45	1.10
$R$	0.981	0.983	0.979



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